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CONTRIBUTION OF MATHEMATICAL CONCEPTS TO MANAGEMENT

Mr. Robert R. Hare, Jr.

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INDUSTRIAL COLLEGE OF THE ARMED FORCES

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Contribution
of
Mathematical Concepts to Management

3 September 1963

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3 September 1963

LT. COLONEL KNIGHT: The atmosphere of the advancing technology and management science is mathematics. Our speaker today has had considerable experience in using mathematics in military operations, research problems and other operations research problems. As you know from his biography, he is the Director of the Operations Research, Incorporated Scientific Support Group, the Army Institute of Advanced Studies, Carlisle Barracks.

Gentlemen, I present to you, Mr. Robert R. Hare, Jr.

MR. HARE: Thank you, Colonel Knight.

The problem of designing or modifying a complex system in response to technological and economic change has become increasingly difficult in today's business and military operations. Associated with this problem is the need to understand the inter-relationships among system elements in order to predict the future behavior of the system and facilitate its control by top management.

It has become apparent in recent years that test operating experience alone is no longer an adequate means for attaining this understanding, especially where it is necessary to project the effect of new conditions on a system. Such experience usually represents only a small sample of the possible ways in which the system might be operated. In certain instances measurements and analysis of past operations alone may reveal simple cause and effect relationships from which predictions can be made regarding the consequences of alternative modes of operation. For most modern systems, however, such predictions cannot be made until after a great of additional information has been collected.

One possible way of getting additional information about a system is by carrying out real system experiments. This concept proposes deliberate experimentation with the operating system itself, or a large segment of it. Its objective is that of any experiment, namely, to gain information otherwise not available, or to test a theory, which, if valid, has broader practical applications. Although real system experimentation has been - and continues to be - carried out by both industry and the Armed Forces, it has certain serious limitations. This approach cannot be used in many instances because of high costs both in money and in time; because of unacceptable consequences in the event of failure; or because the approach may be downright unfeasible.

There is, for example, only a limited amount of useful experiment taken that can be carried out on certain systems related to Civil Defense, or to tactical operations on a battlefield, since such systems only become fully operational in times of emergency or war. Perhaps the most serious limitation of real system experimentation is the fact that it cannot be conducted on proposed future systems that are still only tentative concepts or preliminary designs. This is an area in which improved understanding is especially needed, and which becomes increasingly important as the rate and cost of technological change increases.

The limitations of operating experience and real system experimentation that I have indicated cause systems designers and managers to look for other ways of getting the information they require. Their search has led them to extensive use of system models. The analysis of models, particularly those that are basically mathematical in design, has assumed a vital role as an aid to management in its decisions concerning the development, improvement and operation of major systems.

It is my purpose in this lecture to discuss some of the fundamental characteristics of such models, to indicate some of their advantages and limitations, and describe briefly, by way of example, some of the mathematical techniques that have proved useful in their construction.

A model can be defined as a useful simplified representation of the essentially important aspects of a real object or situation. A model of a system can be a picture, a mechanical or electrical device, a set of mathematical equations, or anything else having characteristics representative of those that are fundamental to the system. Ordinarily, a model will be much simpler than the system it represents, since an important goal of model design is to leave out all detail that appears in the original system, but which plays no significant role in the system's operation. The purpose of this goal is to reduce as much as possible the effort required to analyze the interactions that exist among the different elements of the model.

If the model has been well-designed, such an analysis should then yield information about the corresponding interactions and interrelationships of the original system. In other words, a good model will be designed so that what happens in the model accurately reflects the important things that would happen in the system to which it corresponds. An obvious question at this point is, "How do we know what is important and what is not in a system?" There is no clear-cut answer to this question. Some elements of a system will be of obvious importance; others, not so obvious. Wherever there is a question about the significance of a system element it should be represented in the model, although it may be omitted later if subsequent analysis of the model shows that the questionable element is, in fact, not important.

On the whole, the design of an adequate model is something of an art and requires that the designer have a pretty good feel for the system with which he is concerned. When a model is used one is generally interested in determining quantitative relationships which can then be applied to the system. Therefore, the model must be constructed so that there is a known correspondence between model values and system values. Generally, one is the same or proportional to the other. For example, if a map is a visual model of a certain piece of terrain, distances on the map are proportional to distances on the terrain. The corresponding angles have the same value.

Models are ordinarily classified as "visual," "analog," or "symbolic." Of course, these classifications are not rigid, and several may apply to a given model at the same time. Navigation charts and house floor plans are examples of visual models. You will note that both are much simpler than the things they represent, containing only significant alements of the original, and none of the unessential details. At the same time, they are useful; the charts help the mariner steer from one buoy to the next, even though he's often unable to see more than one buoy at a time. The floor plan helps the architect detect and correct faults in the house layout before starting construction.

A common analog model is the electrical network, which is used to represent mechanical and waterflow systems. The kind of model that has been especially valuable in the study of complex systems is the symbolic model which is composed of mathematical and logical relationships. An example of such a model is one that is used in the engineering design of bridges. I believe Dr. Davidson mentioned this the other day. It is made up of a number of theoretical and empirical relationships involving the mechanics ~~of~~ materials stress analysis and vibration analysis.

With the aid of these mathematical representations bridge structures can be designed quickly and cheaply. Another example of a symbolic model is the computer simulation which has made possible the analysis of complex systems that could not ordinarily be analyzed prior to the development of high-speed digital computers.

I will discuss such models a little more fully in a few minutes.

System models may be employed with regard to a number of objectives. They are generally used to test the implications of changes in a system, and to give us an estimate of the significance of each component with respect to total system operation. The process of constructing a model will, in itself, tend to give one a better overall understanding of the system and how it functions. Information provided by the model can then be used in the design or improvement of the system. It should be noted that model analysis, unlike real system experimentation, can be applied to proposed future systems as well as to those that already exist.

The better overall understanding of system operation provided by a well-constructed model, can always be of great assistance to the manager in designing his management information and control mechanism, and in the development of improved operating policies and procedures. Finally, a bonus effect of many good models has been their value in the training of operating personnel. To illustrate the remarks I've just made, let us consider a model that was developed and used successfully in connection with the water resource management program on the upper Nile.

This slide is a little old; it's about worn out. The Upper Nile System begins at the Aswan Dam in Egypt and includes the main rivers; their dams, reservoirs and tributaries, and the Uganda Lakes. The general objective of water management is for irrigation, power generation, flood control, minimization of evaporation losses, and provision of an adequate flow to the lower Nile. There is a requirement to collect and store enough water during rainy years to carry through a drought cycle. There are also special restrictions on the maximum and minimum permissible water levels in lakes. Evaporation losses can be serious, and must be minimized by keeping water as long as possible in remote mountain reservoirs and lakes. And this increases the complexity of the control problem; for, in order to make a change at

Lake Victoria to effect the water supply at the lower end of the system, some six-weeks lead time is required.

Keeping in mind all of the above requirements and constraints, system management must continually ask such questions as, "What will happen if we do such and such? What are the short-term effects? What are the long-term effects? What would be the effect of constructing a new reservoir at a particular location?" The models shown on the slide helps management answer these questions by providing an integrating logical description of all component elements and their interrelationships, and, by permitting the computation of numerical results relating to the consequences of various possible actions. Actually, the slide shows three models. First there is the map of the Upper Nile System.

Since management was not really interested in the twists and turns of the river, but in the flow that it transmits, and the water losses that occur during flow, the geographic map over on the left was transformed into a kind of hydro-dynamic map, which is right next to the geographic map.

The second model now represents a system in a form reduced to characteristics that are significant with respect to the purpose for which the model was intended. Finally, to permit numerical computations a mathematical model was constructed from the hydro-dynamic map. This third model, represented by the two boxes on the right, and these contain various equations, shows the equations of flow of the system together with such basic constraints as reservoir capacities and permissible water levels.

This model has the flexibility to support top-management planning by developing the consequences of alternative policies or modifications to the system. These consequences may be individually simple events, but in a large system operating over a period of time, they form an intricate chain of circumstances, the estimation of which, is pretty much beyond the capability of any other method.

Before I talk about some specific mathematical techniques I'd like to make several practical observations with regard to the use of system models. First, the usefulness of any model will be limited by the amount and quality of basic operational data available for input. The model designers should ordinarily be able to get most of these data from existing records and measurements of the system's past operation, and, by working closely with system management and operating personnel. However, if information about the value of a particular factor in the system is not available from these sources, a reasonable range of such values may be estimated and the model used to examine system operation for this range. If it is found by this means that the performance of the system is strongly effected by the value of the missing factor, then some experimentation with the system components may be required to attain this value.

Second, it should be emphasized that model analysis cannot be a complete substitute for full-scale trial of a system. Models can help to organize and analyze experience data for the purpose of drawing certain conclusions, but many intangible factors which a model can't take into account will often effect system operation significantly. Therefore, managers must know the limitations of the models they employ, and must use their experience and good judgment to interpret model results in the light of these intangible factors.

This brings up the problem of communication between the mathematical analyst and the manager. A system model may be so intricate in its technical detail that the executive cannot possibly verify the logic to the same extent that he might in a conventional management staff study. I mention the problem, but I know of no way to solve it, except perhaps through the development of mutual confidence over a period of time as each side - the manager and the analyst - makes a conscious effort to understand and appreciate the problems of the other.

Model-building itself is not really new, even though the term is. Models of various types, mathematical and otherwise, have been used for centuries in science and in engineering. The whole purpose of mathematical physics, for example, is to represent the physical laws of nature in the form of mathematical relationships for purposes of analysis. What is fairly new is the greatly increased emphasis placed on the use of models in the area of management decision-making with respect to complex man-machine systems. To meet ever-increasing requirements for models in this area, various mathematical and logical techniques have been developed, which have some general application to many problems encountered in industrial and military operations. These techniques are continually being elaborated and new ones developed as new problems are encountered.

I'd like to use the remainder of this talk to describe briefly some of the more important types of mathematical techniques that have been developed in recent years for use in model construction. First, however, I'd like to mention, in the way of a historical footnote, that the Ancient Babylonians employed mathematics to solve management problems nearly four thousand years ago. In one instance they used arithmetical methods to devise an equitable food-rationing scheme during a period of famine. The records also show that these people made considerable progress in the development and application of elementary geometry and algebra.

It should be noted that one of the major shortcomings of mathematicians in those days was their inability to perform certain arithmetical operations using prime numbers like 7, 11 and 13. This was very embarrassing to them, because everyone from the King on down looked upon mathematicians as being infallible. To cover their ignorance they asserted with great authority that such numbers were endowed with evil magical properties and that any tampering with them would surely have dire consequences. The wisdom and prestige of these learned men was so great that these numbers are still considered to be magical in some quarters, even today.

Although mathematicians are no longer considered to be infallible, they have developed their art for 4,000 years to a point where it is quite useful in many areas of activity. Problem-solving situations that can be treated by mathematical analysis are generally of two types; those whose cause and effect relationships involve a fair degree of certainty, and those in which there is some degree of uncertainty, and which require the use of probability theory and statistics. In addition, there are problem-solving situations that are so complex that mathematical analysis alone cannot be used. These are situations that require computer-simulation techniques.

I shall try to relate the examples that follow, to the categories I've just mentioned. An important problem area in which cause and effect relationships are assumed to be known with some certainty has to do with so-called allocation problems. Allocation problems arise in situations where a system has a number of activities to perform, but where there are restrictions either on the amount of resources, or the way they can be spent, which prevent us from performing each of the activities in the most effective way conceivable. In such situations we wish to allot available resources to the various activities in a way that will optimize the total effectiveness of the system.

An important class of allocation problems is the one in which the effectiveness of the system, and all restrictions, are expressed as linear functions of the allocation variables - and I'll explain what that means in a minute. The analysis of this class of problems is called "linear programming." And it's important, not only because many real situations can be studied this way, but because we have to know how to solve the problem.

On the slide, the symbol lambdus (phonetic) of J is a variable representing the number of units of activity J to be performed. The large sigma in that second expression there means what it usually does; it means the sum of the quantity shown taking over all possible values of J; in this case, over all types of activity. The relations shown are called linear, since no lambdas are multiplied together or appear in powers higher than the first. Basically, there are three kinds of conditions involved in the linear programming problem. First, a negative number of units of an activity cannot be performed. This is represented by that first expression.

Second, allocations are subject to certain practical constraints. Third, there is a linear function of the variables, called the objective, which we wish to optimize. To give you a better feel for the kind of problems solved by linear programming, I'd like to show three classical examples. The first is the simple production problem. In this problem a producer has available limited amounts of several types of resources. Specifically, he has these sub-I units of resource I where I takes a different value for each type of resource.

The producer can make a number of different kinds of commodities using these resources, and he wishes to know how many units of each kind of commodity he should make to maximize his total monetary return. On the slide, the little A sub-I J represents the number of units of resource I required to produce one unit of commodity J. C sub-J is the return per unit of commodity J, and lambdus of J is the number of units that commodity J produced. The constraints are obvious. The producer can't produce a negative number of commodity units, and he can't use more resources than he has available. The problem, then, is to solve for those lambdas of J that maximize the objective; the objective being that bottom equation down there.

The second example is the transportation problem. In this problem there is a given amount of a product located at each of several points of origin. Specifically, there are eight sub-I units of the product located at origin point I. There is also a certain minimum requirement for the product at each of a number of destination points. That is to say, at least B sub-I units of the product must be shipped to the destination J. The given cost of shipping a unit of the product from origin I to destination J is C sub-I J. Lambdus of IJ is the number of units of the product to be shipped from I to J. The shipper wishes to find values of the lambdas that will minimize his total shipping cost. Again, the constraints are obvious. Negative shipments can't be made, nor can more be shipped from an origin point than are located at that origin point. And minimum destination requirements for the product must be met. These three conditions are shown by the first three relationships given.

The third example is the personnel assignment problem. In this problem a group of people have been rated according to their abilities and put into various personnel categories. Little A sub-I is the number of people assigned to personnel category I. People from this group are assigned to openings existing in each of a number of job categories. B sub-J here is the number of job openings to be filled in job category A. C sub-I J is the productivity of a worker in personnel category I who has a job in category J. And lambdus of IJ is the number of workers in category I to be assigned to job category J. The employer wishes to make assignments; that is, determine lambdas so that the total productivity is maximum.

The constraints are that a negative number of workers can't be assigned from a category. And all job openings must be filled. Furthermore, a greater number cannot be assigned than exist in a category.

The applications just cited are only three of a large number of useful applications of linear programming developed over the last few years. These applications include not only the getting up of schedules for the use of an organization's resources, but also theoretical studies to aid policy-makers in determining how activities and resources might be changed to improve the organization's position. The principal limitation of linear programming is the requirement for linearity which excludes many important allocation problems. As I indicated earlier, this requirement was put on to facilitate problem solution, and a number of fairly general methods for solving such problems has been developed.

Some solutions have also been obtained for problems where constraints and/or the objective function are quadratic rather than linear. That is to say, they involve second powers of the variables. A promising development which is still in the beginning stages is dynamic programming. Essentially, this involves the solution of a sequence of allocation problems where the conditions on any problem in the sequence are effected by the solution of all preceding problems in the sequence. The principal difficulty that one must face here is the fact that optimum solution to individual problems won't necessarily optimize the objective function for the sequence taken as a whole.

Let's consider, now, a different type of situation from the ones that I've just discussed. Suppose we have a system that involves one or more input values, and an output value X. Also suppose we observe that for the same choice of input values a different value of X comes out each time the system is operated. This situation can arise if the situation contains chance or Stakhastic (phonetic) elements; and we must consider X to be a random variable. Either by experiments or by analysis we try to find out as much information as possible about the frequency distribution of X. That is, the relative frequency with which different values of X could be expected to occur. From this distribution we may wish to estimate the probability that X will fall within a given range of values on the next trial of the system. We may also wish to condense our distribution of information by expressing it in terms of the mean or expected value of X, together with some value representing the amount of dispersion of X about the mean.

The situation I have just described involves considerations of probability theory and statistics. A part of these considerations is the frequency distribution of the random variable. Frequency distributions are of two types - discrete and continuous. In a discrete distribution the random variable X can take any of a set of discrete values. And the frequency distribution function, as shown on the slide, is a bar graph. The height of each bar shows the probability that X will have a particular value at a given time. The X with the line over it, in the little box, is the expected value of X and is obtained by multiplying each X by its probability of occurrence and then summing these products over all possible values of X.

In a continuous distribution the random variable can take on any of a continuous set of values. And the distribution function, as shown, is a continuous curve. The probability that X will fall within a given range of values is just the area under the curve in this range. This is illustrated on the slide by the shaded portion which is equal in area to the probability that X will fall between 2 and 3 at a given time.

When properly applied, the probability theory is one of the most powerful tools we have today for constructing mathematical models. When improperly applied it can be rather dangerous. Even statisticians have trouble. I like to tell the story about a well-known statistician who confidently waded into a river having an average depth of three feet, and drowned.

I would now like to discuss briefly two of its more important applications - queuing theory and game theory. Queuing theory, or waiting-line theory, has also been referred to as the "Theory of Bottlenecks." Since it deals with problems of congestion and delay I don't think I need to go into any long explanation justifying its importance. Let us consider one of the simplest problems to which the technique has been applied - the tollgate problem. At this point I had a fancy slide showing a tollgate, but somewhere it got lost. So, picture, if you will, a single tollbooth on a turnpike - I think you're all familiar with that - and at this tollbooth we find a line of cars, and there are also cars approaching the tollbooth. The basic components of a queuing situation; customers and a customer input process, service points and a service mechanism, and some sort of queue discipline. Each of these is illustrated in the tollgate problem. The customers are motorists who arrive in some random way at the tollgate which is the service point. And they pay a toll before they may pass this point.

Their queue discipline is to line up on a first-come first-served basis. In this particular problem we assume only one gate. Service times at the tollgate will depend on whether the motorist has his money out, how much change is required, whether or not he stalls his car, and other factors. This summer I had a flat tire right at the tollgate. They wouldn't let me stay.

Both the intervals of time between new arrivals and service time must be treated as random variables. And the frequency distribution must be determined for each of these. In analyzing this sort of queuing problem one is interested in such information as the waiting times of the customers, the number of customers in line, and what fraction of time the servor is idle.

The next slide shows a particular set of conditions, and the probabilities with which there will be various numbers of customers at the tollgate at any given time. In this example an average of six cars per minute arrive at the booth. And the toll collector can handle an average of ten cars per minute. For arrival and service time distributions I have assumed - I won't go into any detail on these - I have assumed what are known as "price-on arrivals," and "exponential service times." These seem to represent fairly well what actually happens in any such situation. It is also assumed that the tollbooth has been in operation long enough so that a steady or routine state of operation has been established. It can be shown in this case that the average number of cars at the gate at any given time is 1½, and the average delay involved is about 9 seconds.

Starting with simple problems like the one just described, queuing theory has been extended and elaborated in many ways. These include such features as multiple servors; multiple waiting lines with special rules as to which one a customer enters; priority of service to certain customers; and the effect of impatient customers who either don't enter a line or leave it before being served. Application of the technique can help managers to design or improve systems in which congestion occurs, by indicating the best number and configuration of serving points and waiting lines, not only for reducing congestion and delay, but also with a view toward overall operating economy.

The factor of economy is an important one. If the system were operated only for the convenience of its customers and enough servors could be provided so that a customer would practically never have to wait in line, such a policy would mean that many servors would stand idle much of the time. And presumably, each servor represents a certain portion of the system's operating cost. To run the system economically, therefore, some balance must be achieved between the cost of customer delay and that of servor idle time.

Queuing theory techniques have been applied to many types of operation, and new applications are being found all the time. Operations that have been analyzed successfully in the past include air, rail and highway traffic management; production line operations; service at store counters; telephone trunking; the maintenance and repair of machines; and a wide variety of military problems.

Game theory is essentially a theory of competition. It deals with situations where the outcome will depend on the decision of two or more participants who have conflicting interests. It is assumed in such situations that each participant will act in a rational manner and will try to benefit as much as possible from his decisions. Most of the practical applications of game theory relate to situations involving two decision-makers - a so-called "Two-Person Game." To illustrate this game, suppose that two players, A and B, - I believe these players were made famous by Gilbert and Sullivan some years ago - suppose these players are competing with one another in a conflict situation.

Furthermore, suppose A has three alternative decisions, or strategies, available to him, and B has four. The first thing each player would do is, make up a table like the one shown on the slide, in which he would estimate the consequences of each possible pairing of strategies selected by himself and his opponent. Suppose the slide shows A's table. Then, the entry A two three represents the value of the game to player A. If he chooses strategy A two and player B chooses strategy B three; the other entries on the table represent the values to A of other strategy combinations. A negative value in the table would indicate a loss by a player - by player A, for the pair of strategies involved.

If each player wins what the other loses, this game is called a "Zero Sum Game." Now, if player B always used the same strategy and A knew what this was, - A can pick any one of the strategies on the side and B can take any of the ones on the top - if player B always uses the same strategy and A knew what this was, A would select that strategy which gained him the most, and he would continue to use it. A would then win the same amount from B each time they played, which would be rather rough on B. Consequently, B must change the strategy he selects each time he plays and do this in some manner unknown to A. To hold his own, A must also change his strategy from play to play. In order to keep the strategies to be used for various plays a secret from one another, A and B will make their selections by some random means. Before doing so, however, each player must determine the relative frequency with which he should select the various strategies open to him, so as to secure for himself the greatest possible expected gain.

A player who changes his strategy from play to play, selecting each new one randomly, but according to prescribed relative frequencies, is said to employ a mixed strategy. His basic problem is to select the optimum mixed strategy. And this is the fundamental problem in game theory.

For those of you who are interested in how A calculates his best mixed strategy, consider the equation shown on the slide. Incidentally, that second equation should be a little A sub-I J, not a big A sub-I J. Let P sub-I be the probability that A will select strategy A sub-I for any given play. And let V by the expected value of the game to A if he uses the best mixed strategy. Then, if the linear equations that are shown on the slide are solved simultaneously for the three Ps - P-1, P-2 and P-3 - the values of these Ps will give the relative frequencies with which the various strategies must be selected, for A to have the optimum mixed strategy. The first equation simply shows that the sum of the probabilities must equal one. That is, A must play some strategy every time.

The second equation states that the expected value of the game to A must be the same no matter which strategy B employs. Player B will determine his best mixed strategy in a similar manner. It should be noted that the expected loss or gain to B must be the same as the expected gain or loss to A. That is, B must have the same value for both player A and B.

Let us look, now, at a particular example of a two-person game. Suppose that a ship is searching a thousand square mile area for a submarine which, in turn, is studiously trying to avoid being found. Furthermore, suppose the search area is divided into two sub-areas, A-1 and A-2, having areas of 300 and 700 square miles, respectively. Finally, suppose that the search rate in A-1 is 10 square miles an hour, while the search rate in A-2 is 30 square miles per hour; and that both hunter and hider know this. A game matrix is set up, as shown on the slide, with detection probability per hour taken as the value of the game to the hunter. This was found by deciding the search rate per hour in a given sub-area, by its area. The problem is to determine what fractions of their time the hunter and hider should spend in each of the sub-areas in order to optimize their respective chances of succeeding.

As it turns out, each has spent 56% of his time in A-1, and 44% of his time in A-2. And the expected probability of detection per hour is about .02. It was interesting to note that if the hunter conducted his search uniformly over the whole thousand square mile area and the hider knew it, the expected detection probability per hour would drop to .01. On the other hand, if the hider moved uniformly over the whole area and the hunter knew it, the expected detection probability per hour would rise to .03.

Game theory is potentially valuable in both business and military situations. Its application in such situations depends largely on how accurately and exhaustively the choices of strategies and their consequences can be determined by the analyst.

Many systems existing today - particularly military systems - are so large and complicated that ordinary means of analyzing them are inadequate. Frequently we can construct mathematical models for portions of such a system, but a study of the system as a whole requires what I like to refer to as the last resort approach;

computer simulation. The basic concept of computer simulation is fairly simple. Think of a system as being composed of a collection of elements together with a set of rules governing the performance and behavior of these elements. These rules will describe not only the behavior of individual elements under various conditions, but also the basic interrelationships among them. Elements of a system can be any object or condition that effect the system's operation. They may be such things as people, machines, equipment, places and environmental conditions. They may even be whole sub-systems whose operations have already been analyzed by another means. The rules of behavior will specify basic element performance. They will prescribe simple cause and effect relationships and they will give frequency distributions for any random variables in the system. And, they will set physical and doctrinal limitations on the behavior of elements.

The rules will also specify decisions as to courses of action to be taken at different stages in the operation of the system. In other words, the first step in constructing a computer simulation model is to examine carefully all the bits and pieces which make up the system and understand their basic relationship to one another. So far, the procedure is really no different than that used to construct any other model; it just takes longer.

The big difference begins with the next step. This is to take each of the elements and rules and represent it somehow in the form of words and instructions for the digital computer. Such representations are then tied together by means of a computer program. This program states in computer language how and in what order the given rules are applied to system elements to produce behavior and interactions similar to those that would occur in an actual system. Such a program is normally written from a logical flow diagram which shows in detail the operating characteristics of the simulated system arranged in their proper order in space and time.

A simulation program is often designed in the form of a large loop. On this particular diagram one of the links in the loop that closes it, got left out. Up at the top there should be an arrow. So, this is not a loop. All elements of the model are examined in a single circuit of the loop and their conditions or states are modified wherever necessary in accordance with the given rules of behavior. Each circuit of the loop simulates the operation of the whole system during one small increment of time. Everything that goes on within that small time is in terms of modifying the elements as considered.

As the computation proceeds the computer acts as nothing more than a large, high-speed bookkeeping machine making and storing all changes in the simulated operation from one time increment to the next, and periodically reading out certain desired information about what is happening. As I have mentioned, the usual purpose in studying a system is to determine whether it can be modified in some way to improve its effectiveness or reduce its cost of operation. This purpose implies the existence of alternative ways of operating, organizing or equipping the system. Therefore, if a computer simulation model is to be used it should be designed to accept all of the various element configurations and characteristics, and all of the operating rules that one might conceivably wish to evaluate.

Furthermore, with the possibility of studying a large number of options, very careful thought should be given to the plan for using the model. For instance, use could be made of experimental design techniques. A pure trial and error approach can turn out to be rather expensive where computer simulation models are involved.

In closing, I'd like to say that the techniques that have been described this morning are only a few examples of what has been and is being accomplished in this field. I hope that these examples, together with the discussion of model-building, in general, have helped you to gain a better practical understanding of the way in which mathematical analysis can assist management in its decision-making role.

Thank you.

QUESTION: Mr. Hare, can you give us any examples where the military has used a combination of these various theories or techniques in the analysis of a major program?

MR. HARE: (This answer may have contained classified information, so it is deleted).

QUESTION: To what level is the battlefield simulation going down - to the Division level, lower or higher? And in what theaters of operation?

MR. HARE: I'm not familiar with all the details of what they're doing right now, but I understand it goes right down to Company level at least. I don't know if they've picked a particular theater of operation or just doing a sort of general-type thing that represents any forces in a particular situation.

QUESTION: Mr. Hare, you mentioned that model complexity often creates a problem for the manager to understand the problem and that it requires his time. Is there some other method; the simplification of some of these models that might be more beneficial than this trying to have people understand each other?

MR. HARE: Well, now, when I said the complexity of the model was liable to leave the manager a little baffled, I was talking about the actual technical details - the mathematical details - involved in coming out with solutions. Now, the manager should be quite familiar with the purpose of the model, what somebody is trying to do with this model, even though he cannot understand all the mathematical detail involved in grinding out answers. He knows what goes in. He knows the purpose of the model, and he can understand what comes out. The secret workings inside are sometimes a little difficult and these he has to take on faith and trust his mathematician.

Now, as to simplifying the model so that he can understand these internal workings, this really defeats the purpose of using mathematics for these sort of analyses, because you would want to take advantage of the most advanced mathematical techniques that are necessary for the solution of the problem. If you can't use these, why, you are not making use of the tools which are available to you. Then you're back in the position of doing essentially the normal staff study. So, unless the manager can spend seven or eight years in college, and then some more years in getting experience in using these things, he can't really be expected to know all of the details of a model.

QUESTION: What is your appraisal, Mr. Hare, of the relative effectiveness of the analytical techniques on the one hand and simulation techniques on the other, considering past efforts and future projected efforts to solve real world problems?

MR. HARE: Well, the analytical techniques, because they have to be in a form that a mathematician can handle analytically. Because, equations will necessarily involve a number of simplifications, approximations, and certain simplifying assumptions, so that he will - in many instances - have to ignore a lot of detail in the situation. Now, in certain situations he can get away with this; he will get answers that will give him at least the order of magnitude solutions to his problem.

In the computer simulation the problem has just gotten so involved that he can't, even with a lot of simplification, study the thing; there are too many parts to it. So then, he goes back and goes into a lot of detail. He just sort of puts together all the pieces and lets the computer do all the recording of information.

So, in a simulation you can probably get a good deal more realistic model of the problem. On the other hand, simulation is extremely expensive; it's extremely time-consuming. Some of these programs for computers - for simulation - may take a year; maybe several years, to prepare. Furthermore, it takes, well, these things spit out bushels of paper with numbers on them, and it takes a great deal of time to analyze the results. You get more information than you really want, even when you're careful.

So that, there are time advantages and money advantages to avoiding the simulation if possible. Does that give you some idea?

LT. COLONEL KNIGHT: Gentlemen, I would like to at this time also acknowledge again the presence of our guest panelists have been provided through the Operations Research Council of Washington, who will be with us in our seminars following this lecture. Mr. Hare will be present in the cafeteria, and there are several old friends and colleagues of his whom he hasn't seen in quite some time. He will be available for some further questioning if you would like to hear him.

Mr. Hare, thank you very much for being with us in this lecture and we were very pleased to have you.